

Renormalization of supersymmetric Yang-Mills theories with soft supersymmetry breaking

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Abstract. The renormalization of supersymmetric Yang-Mills theories with soft supersymmetry breaking is presented using spurion fields for introducing the breaking terms. It is proven that renormalization of the fields and parameters in the classical action yields precisely the correct counterterms to cancel all divergences. In the course of the construction of higher orders additional independent parameters appear, but they can be shown to be irrelevant in physics respects. Thus, the only parameters with influence on physical amplitudes are the supersymmetric and the well-known soft breaking parameters.

1 Introduction

At future experiments at the LHC or at a linear e^+e^- collider, supersymmetric extensions of the standard model can be tested decisively [1]. On the theoretical side, exploiting the potential of these experiments requires a thorough control of the quantization and the renormalization of supersymmetric models. One important characteristic of supersymmetric extensions of the standard model is the appearance of so-called soft supersymmetry-breaking terms [2]. Models with soft-breaking terms have been renormalized using the Wess-Zumino gauge in [3]. The construction in [3] yields a result with an inherent ambiguity. There appear new kinds of parameters that have no interpretation as either supersymmetric or soft-breaking parameters. Hence, it is unclear whether these extra parameters constitute a new kind of free, in principle measurable, input parameters, and how the results would influence the relation to phenomenology. This effect can be understood as a consequence of the construction using a BRS doublet for introducing the soft breaking.

In the present article, an alternative approach to the renormalization of softly broken supersymmetric gauge theories is presented using the spurion fields introduced originally in [2]. Since the spurion fields are supermultiplets by themselves, soft breakings of supersymmetry are distinguished from soft breakings of gauge invariance and other non-standard breakings (see e.g. [4]). Since the spurion fields are dimensionless, they can appear in arbitrary powers in the action – thus in our approach there appear

new parameters, too. We can prove, however, that the additional parameters do not influence physical amplitudes and hence are irrelevant in physics respects.

For the characterization of the symmetries, a Slavnov-Taylor identity of the same structure as in the unbroken case [5, 6] can be used. Since no supersymmetric and gauge invariant regularization is known, we do not rely on the existence of such a scheme and define all Green functions, using the algebraic method, via the Slavnov-Taylor identity. On this basis the relations between the renormalization of soft and supersymmetric parameters, given in [7–10], are not included in the construction; all soft-breaking terms can appear with arbitrary renormalization constants. As demonstrated for supersymmetric QED in [10], a derivation of such results requires a much more sophisticated introduction of the soft-breaking terms and is beyond the pure proof of renormalizability.

We restrict ourselves to a simple, non-Abelian gauge group and exclude spontaneous symmetry breaking and CP violation. Together with the treatment of the intricacies of the standard model due to its spontaneously broken, non-semisimple gauge group [11] and supersymmetric non-abelian [5, 6] and Abelian [12] gauge theories without soft breaking, this should provide the necessary building blocks for the renormalization of the supersymmetric extensions of the standard model.

The outline of the present article is as follows. In Sect. 2 the basic notions of the considered models and of soft supersymmetry breaking are introduced. In Sect. 3 the symmetry identities describing gauge invariance and softly broken supersymmetry are constructed according to the basic idea described above.

Sections 4, 5 constitute the main part of the paper. In Sect. 4 it is shown that – similar to the case of [3] – by

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introducing the external chiral multiplet an infinite number of parameters appears in the most general classical action. That these parameters are all irrelevant in physics respects and do not even appear in practice is demonstrated in Sect. 5. The theorems proven there are our central results and finally also imply that all divergences can be absorbed in accordance with the symmetries. In Sect. 6 our approach is compared to the one of [3] and its advantages and disadvantages are discussed. In the appendix our conventions and the BRS transformations are collected.

2 The model and its symmetries

2.1 Supersymmetric part

We consider supersymmetric Yang-Mills theories with a simple gauge group, coupled to matter. In this class of models there are the following fields:

- One Yang-Mills multiplet in the adjoint representation of the gauge group. This multiplet consists of the spin-1 gauge fields A_a^μ and the spin- $\frac{1}{2}$ gauginos $\lambda_a^\alpha, \bar{\lambda}_{a\dot{\alpha}}$.
- Chiral supersymmetry multiplets (ϕ_i, ψ_i^α) for the matter fields consisting of scalar and spin- $\frac{1}{2}$ fields that transform under a representation of the gauge group which is in general reducible. The corresponding hermitian generators are called T_{ij}^a .

This minimal set of fields corresponds to the Wess-Zumino gauge and is used throughout the whole paper. Still it will be convenient to have the compact superspace notation at hand. In superspace, fermionic variables $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$ and covariant derivatives $D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\theta)_\alpha\partial_\mu, \bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$, are used, and the fields introduced above are combined in the following vector, chiral and field strength superfields¹

$$V_a(x, \theta, \bar{\theta}) = \theta\sigma^\mu\bar{\theta}A_{a\mu}(x) + i\theta\bar{\theta}\bar{\lambda}_a(x) - i\bar{\theta}\theta\lambda_a(x) + \frac{1}{2}\theta\bar{\theta}\bar{\theta}D_a(x), \tag{1}$$

$$\Phi_i(y, \theta) = \phi_i(y) + \sqrt{2}\theta\psi_i(y) + \theta\theta F_i(y), \tag{2}$$

$$W_\alpha = -\frac{1}{8g}\bar{D}\bar{D}(e^{-2gV}D_\alpha e^{2gV}) \tag{3}$$

with the chiral coordinate $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$ and $V = T^a V_a, W_\alpha = T^a W_{a\alpha}$. Whenever we use a superspace expression, it is understood that the auxiliary fields D_a and F_i are eliminated by their respective equations of motion derived from the complete classical action, $\frac{\delta\Gamma_{cl}}{\delta D_a} = \frac{\delta\Gamma_{cl}}{\delta F_i} = \frac{\delta\Gamma_{cl}}{\delta F_i^\dagger} = 0$.

Using this notation and superspace integrals with the normalization $\int d^2\theta\theta\theta = \int d^2\bar{\theta}\bar{\theta}\bar{\theta} = 1$, the supersymmetric part of the classical action reads

$$\Gamma_{\text{susy}} = \int d^4x d^2\theta d^2\bar{\theta} \Phi^\dagger e^{2gV} \Phi + \left(\int d^4x d^2\theta \frac{1}{4} W_\alpha^\alpha W_{a\alpha} + W(\Phi) + h.c. \right) \tag{4}$$

with the superpotential²

$$W(\Phi) = \frac{m_{ij}}{2} \Phi_i \Phi_j + \frac{g_{ijk}}{3!} \Phi_i \Phi_j \Phi_k. \tag{5}$$

2.2 Soft supersymmetry breaking

Soft-breaking terms break supersymmetry without destroying its attractive features. In the present work we restrict the soft-breaking terms to the terms found and classified by Girardello and Grisaru (GG) [2]. Their list of soft-breaking terms is quite short:

- mass terms for scalar fields, $-M_{ij}^2 \phi_i^\dagger \phi_j$,
- holomorphic bilinear and trilinear terms in the scalar fields, $-(B_{ij} \phi_i \phi_j + A_{ijk} \phi_i \phi_j \phi_k + h.c.)$,
- mass terms for gauginos, $\frac{1}{2} (M_\lambda \lambda_a \lambda_a + h.c.)$.

These GG terms have two crucial properties: First, they break supersymmetry without introducing quadratic divergences. And second, they may be viewed as a part of a *power-counting renormalizable and supersymmetric* interaction term with an external supermultiplet (spurion) [2]. This can be shown by introducing one external chiral multiplet with R -weight 0, mass dimension 0 and a constant shift in its \hat{f} component³:

$$\eta(y, \theta) = a(y) + \sqrt{2}\theta\chi(y) + \theta\theta\hat{f}(y), \tag{6}$$

$$\hat{f}(y) = f(y) + f_0. \tag{7}$$

Then the supersymmetric extensions of the above soft breaking terms can easily be written in superspace:

$$\Gamma_{\text{soft}} = - \int d^4x d^2\theta d^2\bar{\theta} \tilde{M}_{ij}^2 \eta^\dagger \eta \Phi_i^\dagger (e^{2gV} \Phi)_j - \int d^4x d^2\theta (\tilde{B}_{ij} \eta \Phi_i \Phi_j + \tilde{A}_{ijk} \eta \Phi_i \Phi_j \Phi_k) + h.c. - \int d^4x d^2\theta \frac{1}{2} \tilde{M}_\lambda \eta W_\alpha^\alpha W_{a\alpha} + h.c. \tag{8}$$

As long as η and its component fields are treated as external fields with arbitrary values, these interaction terms are manifestly supersymmetric. Only in the limit

$$a(x) = \chi(x) = f(x) = 0, \quad \eta(x, \theta) = \theta\theta f_0, \tag{9}$$

they reduce to the soft breaking terms with $\tilde{M}_{ij}^2 |f_0|^2 = M_{ij}^2, \tilde{B}_{ij} f_0 = B_{ij}, \tilde{A}_{ijk} f_0 = A_{ijk}, \tilde{M}_\lambda f_0 = M_\lambda$.

The GG soft breaking terms comprise all possible terms of mass dimension 2 but not all possible terms of mass dimension 3. Obviously, not only $\lambda\lambda$ and $\phi\phi\phi$ but also $\psi\psi$ and $\phi^\dagger\phi\phi$ are supersymmetry-breaking terms of

² Gauge singlets are excluded

³ The \hat{f} component of this external chiral superfield need not be eliminated since \hat{f} is no dynamical field and does not satisfy particular equations of motion

¹ For the vector superfield, the Wess-Zumino gauge is used

mass dimension 3⁴. The terms of the form $\psi\psi$ and $\phi^\dagger\phi\phi$ are excluded from the GG class because in general they introduce quadratic divergences. However, as mentioned e.g. in [4], in many concrete models, like the minimal supersymmetric extension of the standard model, these quadratic divergences are absent. Therefore, concerning only the quadratic divergences, the GG class is too narrow.

If soft breaking is introduced via the coupling to η , the non-GG terms are excluded, since they cannot be extended to a power-counting renormalizable and supersymmetric interaction such as in (8). This means that the possible supersymmetric coupling to the spurion η is the more profound characterization of the GG soft breaking terms than absence of quadratic divergences.

3 Quantization

3.1 Construction of the Slavnov-Taylor identity

Our aim is now to find a definition of supersymmetric gauge theories with soft breaking. Analogously to the case without soft breaking, softly broken supersymmetry should be combined with gauge invariance in a single Slavnov-Taylor identity. Since soft breaking terms are characterized by the possible coupling to the external η multiplet, there is the following possibility: The Slavnov-Taylor identity has the same form as in the unbroken case but it contains also the η multiplet. In this way, first a fully supersymmetric model is described. Then η is set to the constant (9), and in this way the soft breaking is introduced.

According to this approach, the Slavnov-Taylor identity is constructed along the same lines as in the unbroken case [6]. The basic elements of the construction are the following: First, BRS transformations are introduced at the classical level. Since supersymmetry, gauge transformations, and translations are deeply entangled in the Wess-Zumino gauge, all three symmetries have to be combined in the BRS transformations s , and three kinds of ghost fields have to be used. These are the fields

$$c_a(x), \epsilon^\alpha, \bar{\epsilon}^{\dot{\alpha}}, \omega^\nu, \quad (10)$$

corresponding to gauge and supersymmetry transformations and translations, respectively. Only the Faddeev-Popov ghosts c_a are quantum fields, whereas the other ghosts are space-time independent constants because the

⁴ For instance, in the case of the minimal supersymmetric standard model the $\phi\phi\phi$ GG terms are (we adopt the conventions of [4])

$$m_{10}\lambda_t H_2 Q\bar{t} + m_8\lambda_b H_1 Q\bar{b} + m_6\lambda_\tau H_1 L\bar{\tau},$$

whereas the following non-GG terms are also perfectly gauge-invariant supersymmetry-breaking terms that do not induce quadratic divergences:

$$m_9\lambda_t H_1^* Q\bar{t} + m_7\lambda_b H_2^* Q\bar{b} + m_5\lambda_\tau H_2^* L\bar{\tau}.$$

corresponding symmetries are global. The statistics of all ghost fields is opposite to the one required by the spin-statistics theorem. The explicit form of the BRS transformations can be found in the appendix.

Second, the sum of the gauge fixing and ghost terms has to be BRS invariant in order to ensure the decoupling of the unphysical degrees of freedom and the unitarity of the physical S-matrix. Thus it can be obtained as the BRS transformation of some fermionic expression with ghost number -1 . In order to define such an expression we introduce the antighosts $\bar{c}_a(x)$ and auxiliary fields B_a and write the usual renormalizable gauge fixing term with arbitrary gauge parameter ξ and a linear gauge fixing function $f_a = \partial_\mu A_a^\mu$ as

$$\Gamma_{\text{fix, gh}} = \int d^4x s \left[\bar{c}_a (f_a + \frac{\xi}{2} B_a) \right]. \quad (11)$$

Third, most of the BRS transformations are non-linear in the propagating fields and thus affected by quantum corrections. In order to cope with the renormalization of the composite operators $s\varphi_i$ we couple them to external fields Y_i :

$$\Gamma_{\text{ext}} = \int d^4x \left(Y_{A_a^\mu} s A_a^\mu + Y_{\lambda_a^\alpha} s \lambda_{a\alpha} + Y_{\bar{\lambda}_{a\dot{\alpha}}} s \bar{\lambda}_{a\dot{\alpha}} + Y_{\phi_i} s \phi_i + Y_{\phi_i^\dagger} s \phi_i^\dagger + Y_{\psi_i^\alpha} s \psi_{i\alpha} + Y_{\bar{\psi}_{i\dot{\alpha}}} s \bar{\psi}_{i\dot{\alpha}} + Y_{c_a} s c_a \right). \quad (12)$$

Note that the implicit elimination of the D_a and F_i, F_i^\dagger fields yields additional bilinear terms in the external Y fields. Using the external Y fields we can write down the Slavnov-Taylor operator $S(\cdot)$ corresponding to the BRS operator s . Acting on a general functional \mathcal{F} it reads:

$$S(\mathcal{F}) = S_0(\mathcal{F}) + S_{\text{soft}}(\mathcal{F}), \quad (13)$$

$$\begin{aligned} S_0(\mathcal{F}) = \int d^4x & \left(\frac{\delta\mathcal{F}}{\delta Y_{A_a^\mu}} \frac{\delta\mathcal{F}}{\delta A_a^\mu} + \frac{\delta\mathcal{F}}{\delta Y_{\lambda_a^\alpha}} \frac{\delta\mathcal{F}}{\delta \lambda_a^\alpha} + \frac{\delta\mathcal{F}}{\delta Y_{\bar{\lambda}_{a\dot{\alpha}}}} \frac{\delta\mathcal{F}}{\delta \bar{\lambda}_{a\dot{\alpha}}} \right. \\ & + \frac{\delta\mathcal{F}}{\delta Y_{\phi_i}} \frac{\delta\mathcal{F}}{\delta \phi_i} + \frac{\delta\mathcal{F}}{\delta Y_{\phi_i^\dagger}} \frac{\delta\mathcal{F}}{\delta \phi_i^\dagger} + \frac{\delta\mathcal{F}}{\delta Y_{\psi_i^\alpha}} \frac{\delta\mathcal{F}}{\delta \psi_i^\alpha} \\ & + \frac{\delta\mathcal{F}}{\delta Y_{\bar{\psi}_{i\dot{\alpha}}}} \frac{\delta\mathcal{F}}{\delta \bar{\psi}_{i\dot{\alpha}}} + \frac{\delta\mathcal{F}}{\delta Y_{c_a}} \frac{\delta\mathcal{F}}{\delta c_a} + s\bar{c}_a \frac{\delta\mathcal{F}}{\delta \bar{c}_a} + sB_a \frac{\delta\mathcal{F}}{\delta B_a} \Big) \\ & + s\epsilon^\alpha \frac{\partial\mathcal{F}}{\partial \epsilon^\alpha} + s\bar{\epsilon}_{\dot{\alpha}} \frac{\partial\mathcal{F}}{\partial \bar{\epsilon}_{\dot{\alpha}}} + s\omega^\nu \frac{\partial\mathcal{F}}{\partial \omega^\nu}, \end{aligned} \quad (14)$$

$$\begin{aligned} S_{\text{soft}}(\mathcal{F}) = \int d^4x & \left(sa \frac{\delta\mathcal{F}}{\delta a} + sa^\dagger \frac{\delta\mathcal{F}}{\delta a^\dagger} + s\chi^\alpha \frac{\delta\mathcal{F}}{\delta \chi^\alpha} \right. \\ & + s\bar{\chi}_{\dot{\alpha}} \frac{\delta\mathcal{F}}{\delta \bar{\chi}_{\dot{\alpha}}} + sf \frac{\delta\mathcal{F}}{\delta f} + sf^\dagger \frac{\delta\mathcal{F}}{\delta f^\dagger} \Big). \end{aligned} \quad (15)$$

Only the linear BRS transformations appear explicitly here.

3.2 Defining symmetry identities

Now we are in the position to spell out the complete definition of the symmetries of the model as a set of requirements on the effective action Γ , the quantum extension

Table 1. Quantum numbers. R, Q_c, GP, dim denote R -weight and ghost charge, Grassmann parity and the mass dimension, respectively. The R -weights n_i of the chiral multiplets are left arbitrary. The quantum numbers of the external fields Y_i introduced in Sect. 3 can be obtained from the requirement that Γ_{ext} is neutral, bosonic and has $dim = 4$. The commutation rule for two general fields is $\chi_1\chi_2 = (-1)^{GP_1GP_2}\chi_2\chi_1$

χ	A_a^μ	λ_a^α	ϕ_i	ψ_i^α	a	χ^α	\hat{f}	c_a	ϵ^α	ω^ν	\bar{c}_a	B_a
R	0	1	n_i	$n_i - 1$	0	-1	-2	0	1	0	0	0
Q_c	0	0	0	0	0	0	0	+1	+1	+1	-1	0
GP	0	1	0	1	0	1	0	1	0	1	1	0
dim	1	3/2	1	3/2	0	1/2	1	0	-1/2	-1	2	2

of the classical action Γ_{cl} and the generating functional of one-particle irreducible vertex functions:

– Slavnov-Taylor identity:

$$S(\Gamma) = 0. \quad (16)$$

– Gauge fixing condition:

$$\frac{\delta\Gamma}{\delta B_a} = \frac{\delta\Gamma_{\text{fix}}}{\delta B_a} = f_a + \xi B_a. \quad (17)$$

– Translational ghost equation:

$$\frac{\delta\Gamma}{\delta\omega^\nu} = \frac{\delta\Gamma_{\text{ext}}}{\delta\omega^\nu} = \int d^4x \sum_{\varphi_i} (-1)^{GP_i} Y_i i\partial_\nu \varphi_i \quad (18)$$

with Γ_{ext} in (12), and where (φ_i, Y_i) runs over the dynamical fields with corresponding Y fields and GP_i denotes the Grassmann-parity of φ_i .

– Global symmetries: We require Γ to be invariant under CP conjugation and under global gauge transformations and continuous R -transformations and to preserve ghost number (see Table 1). There may be further symmetries such as lepton number conservation, but these we leave unspecified. We only assume that the global symmetries exclude mixings between the ψ_i and the λ_a , between ϕ_i and ϕ_j^\dagger and between the combinations $\hat{f}\phi_i$ and $(\hat{f}\phi_j)^\dagger$.

– Physical part: As already stated in Sect. 2.2, the physical part of the effective action is defined to be

$$\Gamma|_{a=\chi=f=0}. \quad (19)$$

In this limit, already defined in (9), supersymmetry is softly broken by GG terms.

For later use we introduce the abbreviation $\text{Sym}(\Gamma) = 0$ for this set of symmetry requirements:

$$\text{Sym}(\Gamma) = 0 \Leftrightarrow (16), (17), (18), \text{Global symmetries}. \quad (20)$$

The canonically normalized classical action is given by the sum

$$\Gamma_{\text{cl, canonical}} = \Gamma_{\text{susy}} + \Gamma_{\text{soft}} + \Gamma_{\text{fix, gh}} + \Gamma_{\text{ext}}, \quad (21)$$

with eliminated D_a and F_i fields. The construction guarantees that $\text{Sym}(\Gamma_{\text{cl, canonical}}) = 0$. Its explicit form reads

$$\Gamma_{\text{cl, canonical}}|_{a=\chi=0} = \Gamma_{\text{susy}}^0 + \Gamma_{\text{soft}}^0 + \Gamma_{\text{fix, gh}}^0 + \Gamma_{\text{ext}}^0 + \Gamma_{\text{bil}}^0, \quad (22)$$

$$\begin{aligned} \Gamma_{\text{susy}}^0 = & \int d^4x \left(-\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{i}{2}\bar{\lambda}^a\bar{\sigma}^\mu(D_\mu\lambda)^a \right. \\ & + \frac{i}{2}\lambda^a\sigma^\mu(D_\mu\bar{\lambda})^a + (D^\mu\phi)^\dagger(D_\mu\phi) + \bar{\psi}\bar{\sigma}^\mu iD_\mu\psi \\ & - \sqrt{2}g(i\bar{\psi}\lambda\phi - i\phi^\dagger\lambda\psi) \\ & - \left. \left(\frac{1}{2}\psi_i\psi_j \frac{\partial^2 W(\phi)}{\partial\phi_i\partial\phi_j} + h.c. \right) \right. \\ & \left. - \frac{1}{2}(\phi^\dagger gT^a\phi)^2 - \left| \frac{\partial W(\phi)}{\partial\phi_i} \right|^2 \right), \quad (23) \end{aligned}$$

$$\begin{aligned} \Gamma_{\text{soft}}^0 = & \int d^4x \left(-\tilde{M}_{ij}^2 \hat{f}^\dagger \hat{f} \phi_i^\dagger \phi_j \right. \\ & - \left(\tilde{B}_{ij} \hat{f} \phi_i \phi_j + \tilde{A}_{ijk} \hat{f} \phi_i \phi_j \phi_k + h.c. \right) \\ & \left. + \frac{1}{2} \left(\tilde{M}_\lambda \hat{f} \lambda^a \lambda^a + h.c. \right) \right), \quad (24) \end{aligned}$$

$$\Gamma_{\text{fix, gh}}^0 = \int d^4x \left(B_a f_a + \frac{\xi}{2} B_a^2 \right) + \Gamma_{\text{gh}}^0, \quad (25)$$

$$\begin{aligned} \Gamma_{\text{gh}}^0 = & \int d^4x \left(-\bar{c}_a \partial_\mu (D^\mu c)_a - \bar{c}_a \partial^\mu (i\epsilon\sigma_\mu\bar{\lambda}_a - i\lambda_a\sigma_\mu\bar{\epsilon}) \right. \\ & \left. + \xi i\epsilon\sigma^\nu\bar{\epsilon}(\partial_\nu\bar{c}_a)\bar{c}_a \right), \quad (26) \end{aligned}$$

$$\Gamma_{\text{ext}}^0 = \Gamma_{\text{ext}}|_{\substack{D_a \rightarrow -g\phi^\dagger T_a \phi \\ F_i \rightarrow -(\partial W(\phi)/\partial\phi_i)}}, \quad (27)$$

$$\Gamma_{\text{bil}}^0 = \int d^4x \left(\frac{1}{2}(Y_{\lambda_a}\epsilon + Y_{\bar{\lambda}_a}\bar{\epsilon})^2 + 2(Y_{\psi_i}\epsilon)(Y_{\bar{\psi}_i}\bar{\epsilon}) \right). \quad (28)$$

As indicated by the superscript 0 , the part containing the external a and χ fields is suppressed here because its concrete form is not relevant for our discussion, and only the \hat{f} component of the η multiplet is retained. Furthermore, we have introduced the gauge covariant derivative

$$D_\mu = \partial_\mu + igT^a A_a^\mu, \quad (29)$$

where in the adjoint representation T^a has to be replaced by $-if^{abc}$ defined by $[T^a, T^b] = if^{abc}T^c$, and the field strength tensor

$$igT^a F_a^{\mu\nu} = [D^\mu, D^\nu], \quad (30)$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - gf^{abc}A_b^\mu A_c^\nu. \quad (31)$$

More general classical solutions of the symmetry requirements will be given Sect. 4.1.

4 Renormalization I: Basics

The symmetry identities constitute a rigorous definition of the considered models. However, it remains to be checked

whether the models defined in this way are renormalizable. In the present section the usual analysis of the structure of the symmetric counterterms is applied, and the existence of infinitely many different types of symmetric counterterms is found. The role of these counterterms will be discussed in Sect. 5.

4.1 Generalized classical solution

In this subsection we assume that the symmetry identities can be established at each order by adding appropriate counterterms. Once the symmetries hold at the order \hbar^n , there still may arise divergences and counterterms may be added. Both the divergences and the counterterms cannot interfere with the symmetries, which means that both are of the form Γ_{sym} with

$$\begin{aligned} & \text{Sym}(\Gamma_{\leq n\text{-Loop, regularized}} + \hbar^n \Gamma_{\text{sym}}) \\ &= \text{Sym}(\Gamma_{\leq n\text{-Loop, regularized}}) + \mathcal{O}(\hbar^{n+1}), \end{aligned} \quad (32)$$

which reduces to

$$\text{Sym}(\Gamma_{\text{cl}} + \zeta \Gamma_{\text{sym}}) = \mathcal{O}(\zeta^2), \quad (33)$$

with some arbitrary infinitesimal parameter ζ , since all symmetry identities are linear or bilinear in Γ . Γ_{cl} is the classical action, i.e. $\Gamma = \Gamma_{\text{cl}} + \mathcal{O}(\hbar)$.

A model is renormalizable if all divergences can be absorbed by counterterms corresponding to renormalization of the fields and parameters in the classical action and if the number of physical parameters is finite.

Equation (33) shows how to find the general structure of the possible divergences and counterterms. Since the perturbed action $\Gamma_{\text{cl}} + \zeta \Gamma_{\text{sym}}$ is a solution of the symmetry identities in terms of a local power-counting renormalizable functional (classical solution), simply the most general of these classical solutions has to be calculated.

In this subsection we determine a certain set of classical solutions with a result reminding of the result of [3]. Beyond the supersymmetric and soft breaking parameters there appear new kinds of free parameters. In fact, our solutions depend on infinitely many parameters!

One way to obtain classical solutions different from $\Gamma_{\text{cl, canonical}}$ in (21) is obvious. Since η is neutral with respect to all quantum numbers and has dimension 0 it can appear without any restrictions in the classical action. Indeed,

$$\begin{aligned} \Gamma_{\text{susy}} + \Gamma_{\text{soft}} &= \int d^4x d^2\theta d^2\bar{\theta} r_{1ij}(\eta, \eta^\dagger) \Phi_i^\dagger (e^{2gV} \Phi)_j \\ &+ \int d^4x d^2\theta \left(r_2(\eta) W_a^\alpha W_{a\alpha} \right. \\ &\left. - r_{3ij}(\eta) \Phi_i \Phi_j - r_{4ijk}(\eta) \Phi_i \Phi_j \Phi_k \right) + h.c. \end{aligned} \quad (34)$$

is a possible generalization of (4), (8) that maintains all symmetry properties of Γ_{cl} . Here r_1 is an arbitrary real function of η, η^\dagger , and r_2, r_3, r_4 are holomorphic functions

of η . Expanding $r_1 \dots r_4$ in a Taylor series leads to infinitely many interaction terms in Γ_{cl} . The fact that this generalized action is still symmetric means that to all of these terms there can be divergent loop contributions and that to each of them a normalization condition is needed.

There is a further, more complicated way to perturb a classical solution of the symmetry requirements. We can modify the superfields appearing in Γ_{susy} and Γ_{soft} by terms depending on a, χ, f . If these modifications are accompanied by suitable changes in the BRS transformations in Γ_{ext} , again classical solutions are obtained. One specific possibility is the following modification of the chiral superfields:

$$\begin{aligned} \Phi_i &= u_{1ij}(a, a^\dagger) \phi_j + \sqrt{2}(u_1 u_2)_{ij}(a, a^\dagger) \theta \psi_j \\ &- \sqrt{2}(u_1 u_3)_{ij}(a, a^\dagger) \theta \chi \phi_j + \theta \theta F_i, \end{aligned} \quad (35)$$

where this modification is parametrized by three arbitrary functions u_1, u_2, u_3 of a and a^\dagger . These fields Φ_i transform as chiral superfields if the BRS transformations and thus Γ_{ext} is redefined as

$$\begin{aligned} \Gamma_{\text{ext}}^{\phi, \psi\text{-Part}} &= \int d^4x \left(Y_{\phi_i} \left[\sqrt{2} u_{2ij} \epsilon \psi_j - (u_1^{-1} s_\epsilon u_1)_{ij} \phi_j \right. \right. \\ &- \sqrt{2} u_{3ij} \epsilon \chi \phi_j \left. \right] - Y_{\psi_i \alpha} \left[-(u_2^{-1} u_1^{-1} s_\epsilon u_1 u_2)_{ij} \psi_j^\alpha \right. \\ &+ \sqrt{2} (u_2^{-1} u_3 u_2)_{ij} \epsilon \psi_j \chi^\alpha - \sqrt{2} (u_2^{-1} u_3 u_3)_{ij} \epsilon \chi \phi_j \chi^\alpha \\ &+ (u_2^{-1} u_1^{-1} (s_\epsilon u_1 u_3) - u_2^{-1} u_3 u_1^{-1} (s_\epsilon u_1))_{ij} \phi_j \chi^\alpha \\ &- \sqrt{2} i (\bar{\epsilon} \sigma^\mu)^\alpha u_{2ij}^{-1} (D_\mu \phi_j \\ &+ (u_1^{-1} \partial_\mu u_1)_{jk} \phi_k + u_{3jk} \phi_k \partial_\mu a) \\ &+ \sqrt{2} \epsilon^\alpha (u_1 u_2)_{ij}^{-1} F_j + \sqrt{2} \epsilon^\alpha (u_2^{-1} u_3)_{ij} \phi_j \dot{f} \left. \right] \\ &+ h.c. + \text{Terms involving } c, \omega^\nu \left. \right). \end{aligned} \quad (36)$$

Here s_ϵ denotes only the $\epsilon, \bar{\epsilon}$ -dependent part of the BRS transformation. The terms involving c, ω^ν are identical to those in (12). Using Φ_i from (35) in $\Gamma_{\text{susy, soft}}$ together with the redefined Γ_{ext} , we obtain a further set of classical solutions.

Analogously, the vector superfield and the corresponding part of Γ_{ext} can be modified as follows:

$$\begin{aligned} V &= v_1(a, a^\dagger) \left(\theta \sigma^\mu \bar{\theta} A_\mu \right. \\ &+ i \theta \bar{\theta} \bar{\theta} (\bar{\lambda} v_2(a, a^\dagger) + \bar{\sigma}^\mu \chi A_\mu v_3(a, a^\dagger)) \\ &- i \bar{\theta} \bar{\theta} \theta (\lambda v_2(a, a^\dagger) - \sigma^\mu \bar{\chi} A_\mu v_3(a, a^\dagger)) \left. \right) \\ &+ \frac{1}{2} \theta \bar{\theta} \bar{\theta} D, \end{aligned} \quad (37)$$

$$\begin{aligned} \Gamma_{\text{ext}}^{A_\mu, \lambda\text{-Part}} &= \int d^4x \left(Y_{A_{a\mu}} \left[i \epsilon \sigma_\mu (\bar{\lambda} v_2 + \bar{\sigma}^\nu \chi A_{a\nu} v_3) \right. \right. \\ &- i (\lambda_a v_2 + \bar{\chi} \bar{\sigma}^\nu A_{a\nu} v_3) \sigma_\mu \bar{\epsilon} - A_{a\mu} (v_1^{-1} s_\epsilon v_1) \left. \right] \\ &+ \left(-Y_{\lambda_a \alpha} \left[\frac{i}{2} \epsilon \sigma^{\rho\sigma} (v_1 v_2)^{-1} F_{a\rho\sigma} (v_1 A) \right. \right. \end{aligned}$$

$$\begin{aligned}
 & + i(v_1 v_2)^{-1} \epsilon D_a + \sqrt{2} v_3 v_2^{-1} \hat{f}^\dagger \bar{\epsilon} \bar{\sigma}^\mu A_{a\mu} \\
 & - (v_1^{-1} v_2^{-1} s_\epsilon v_1 v_2) \lambda_\alpha - [i \epsilon \sigma_\mu (\bar{\lambda}_\alpha + v_3 v_2^{-1} \bar{\sigma}^\nu \chi A_{a\nu}) \\
 & - i(\lambda + v_3 v_2^{-1} \bar{\chi} \bar{\sigma}^\nu A_{a\nu}) \sigma_\mu \bar{\epsilon}] \bar{\chi} \bar{\sigma}^\mu v_3 \\
 & - v_3 v_2^{-1} \sqrt{2} i \epsilon \sigma^\nu (\partial_\nu a^\dagger) \bar{\sigma}^\mu A_{a\mu} \\
 & - (s_\epsilon v_3) v_2^{-1} \bar{\chi} \bar{\sigma}^\mu A_{a\mu} \Big] + h.c. \Big) \\
 & + \text{Terms involving } c, \omega^\nu \Big) . \tag{38}
 \end{aligned}$$

Here a modified field strength tensor $F_{a\rho\sigma}(v_1 A) = \partial_\rho(v_1 A_{a\sigma}) - \partial_\sigma(v_1 A_{a\rho}) - g f^{abc} v_1^2 A_{b\rho} A_{c\sigma}$ has been introduced.

Note that the functions u_1, u_2, v_1, v_2 are a, a^\dagger -dependent generalizations of field renormalizations of the matter and gauge fields. On the other hand, u_3, v_3 are new kinds of parameters corresponding to field renormalizations of the form

$$\psi \rightarrow \psi - u_3 \chi \phi , \tag{39}$$

$$\lambda_\alpha \rightarrow \lambda_\alpha - v_3 (\sigma^\mu \bar{\chi})_\alpha A_\mu . \tag{40}$$

In addition to these modifications, obviously a field renormalization of the Faddeev-Popov ghost

$$c \rightarrow \sqrt{Z_c} c , \quad Y_c \rightarrow \sqrt{Z_c}^{-1} Y_c \tag{41}$$

and renormalization of all parameters appearing in Γ_{cl} in (21) is possible without violating the symmetry identities.

We conclude that the supersymmetry algebra is unstable in the sense that it allows for arbitrary functions $u_{1,2,3}$ and $v_{1,2,3}$ with an infinite number of Taylor coefficients that have to be renormalized. So, even without calculating the classical solution to the symmetry identities in full generality, we know that infinitely many normalization conditions are needed and the effective action Γ depends on infinitely many parameters.

In the physical limit $a = \chi = f = 0$ or already in the limit $a = \chi = 0$, the functions r_i, u_i, v_i reduce to usual field renormalizations and two additional parameters $u_3(0), v_3(0)$. Taking these two parameters into account, the canonically normalized classical action $\Gamma_{\text{cl, canonical}}$ in (22) changes as follows:

$$\begin{aligned}
 \Gamma_{\text{cl, canonical}}|_{a=\chi=0} & = \Gamma_{\text{cl, canonical}}|_{a=\chi=0}^{(22)} \\
 & + \int d^4 x \left(-Y_{\psi i \alpha} (\sqrt{2} \epsilon^\alpha \hat{f} u_{3ij}(0) \phi_j) \right. \\
 & \left. - Y_{\lambda_\alpha \alpha} \sqrt{2} v_3(0) \hat{f}^\dagger \bar{\epsilon} \bar{\sigma}^{\mu\alpha} A_{a\mu} + h.c. \right) . \tag{42}
 \end{aligned}$$

Only the external field part is influenced by the new parameters.

4.2 Remarks on anomalies

In the preceding subsection we have assumed that the symmetry identities can be maintained at each order of perturbation theory. In principle this need not be true,

because there could be anomalies. For unbroken supersymmetric Yang-Mills theories it is well known that the only possible anomaly is the supersymmetric extension of the chiral gauge anomaly [15, 5, 6]. In particular, the relevant cohomology does not depend on the chiral multiplets at all. In spite of the soft breaking, the formulation of our model is the same as the one for unbroken supersymmetric Yang-Mills theories except for the appearance of the additional chiral η multiplet of dimension 0. Therefore, we assume that our model is anomaly free and the symmetry identities can be restored by suitable counterterms at each order.

However, one also has to check for infrared anomalies, i.e. breakings of the symmetry identities that can only be absorbed by counterterms of infrared dimension less than 4. Using the assignments from [3]⁵, in principle counterterms of infrared dimension ≥ 2.5 could show up. However, there are no such counterterms of infrared dimension < 4 that involve at least two propagating fields. The other ones cannot be inserted in higher order loop diagrams and thus are harmless, so there are no infrared anomalies.

5 Renormalization II: Physical part of the model

In general, a model depending on an infinite number of parameters has no predictive power. But this is not necessarily the case here, because all physical amplitudes have to be derived from the effective action Γ in the limit (9), $a = \chi = f = 0$. And we have not yet checked which of the parameters can have any influence on Γ in this limit.

In this section we prove two theorems showing that the infinitely many unwanted parameters are irrelevant for physical quantities and do not appear in practical calculations. Thus the number of physical parameters is finite and the considered models are renormalizable. And moreover, the set of physical parameters can be identified with the supersymmetric and soft breaking parameters.

The essentials of the two theorems are the following:

1. The only quantities $\Gamma|_{a=\chi=Y_i=0}$, i.e. Green functions without external a, χ or Y_i fields, depend on are
 - the field renormalization constants $Z_A, Z_\lambda, Z_c, Z_\phi, Z_\psi$,
 - the gauge coupling g ,
 - the parameters in the superpotential m_{ij}, g_{ijk} ,
 - the soft breaking parameters $\tilde{M}_{ij}^2, \tilde{B}_{ij}, \tilde{A}_{ijk}, \tilde{M}_\lambda$.
 More details and the proof can be found in Sect. 5.2.
2. In practical calculations it is sufficient to solve the symmetry identities in the limit $a = \chi = 0$,

$$\text{Sym}(\Gamma)|_{a=\chi=0} = 0 . \tag{43}$$

Each of these solutions can be extended to a full solution Γ^{exact} that contains the same physics and satisfies

$$\text{Sym}(\Gamma^{\text{exact}}) = 0 , \tag{44}$$

$$\Gamma|_{a=\chi=0} = \Gamma^{\text{exact}}|_{a=\chi=0} . \tag{45}$$

⁵ For the spurion field components we use $\dim_{\text{IR}}(a) = 2$, $\dim_{\text{IR}}(\chi, f) = 1$

Since in the evaluation of $\text{Sym}(\Gamma)|_{a=\chi=0}$ the unphysical parameters do not appear one has no need to calculate Feynman rules or vertex functions involving these parameters. This theorem is proven in Sect. 5.1 for the classical level and Sect. 5.3 for the quantum level.

For practical calculations the theorems have an important implication. It is a possible and sufficient prescription to impose only $\text{Sym}(\Gamma)|_{a=\chi=0} = 0$ and require normalization conditions only for the physical parameters listed in theorem 1. Each solution of this prescription is equivalent in physics respects to a full solution of the symmetry identities, and any two solutions differ only in the physically irrelevant part.

The proofs of these theorems are now given in the order of their logical interdependence. First we prove a lemma which is a more general form of theorem 2 on the classical level and introduce some useful notation. Then this lemma is used to prove theorem 1 and finally theorem 2 on the quantum level.

5.1 Classical solution and invariant counterterms

Let R be the following operator for a renormalization transformation of all parameters and fields appearing in $\Gamma_{\text{cl, canonical}}|_{a=\chi=0}$ defined in (42):

$$\begin{aligned}
R : \\
\{A^\mu, Y_{A^\mu}, \rightarrow \{\sqrt{Z_A} A^\mu, \sqrt{Z_A}^{-1} Y_{A^\mu}, \\
B, \bar{c}, \xi\} \quad \sqrt{Z_A}^{-1} B, \sqrt{Z_A}^{-1} \bar{c}, Z_A \xi\} \\
\{\lambda, Y_\lambda\} \rightarrow \{\sqrt{Z_\lambda} \lambda, \sqrt{Z_\lambda}^{-1}\} \\
\{c, Y_c\} \rightarrow \{\sqrt{Z_c} c, \sqrt{Z_c}^{-1} Y_c\} \\
\{\phi_i, Y_{\phi_i}\} \rightarrow \{\sqrt{Z_{\phi_{ij}}} \phi_j, \sqrt{Z_{\phi_{ij}}}^{-1} Y_{\phi_j}\} \\
\{\psi_i, Y_{\psi_i}\} \rightarrow \{\sqrt{Z_{\psi_{ij}}} \psi_j, \sqrt{Z_{\psi_{ij}}}^{-1} Y_{\psi_j}\} \\
\{g, m_{ij}, g_{ijk}\} \rightarrow \{g + \delta g, m_{ij} + \delta m_{ij}, g_{ijk} + \delta g_{ijk}\} \\
\{\tilde{M}_{ij}^2, \tilde{B}_{ij}, \rightarrow \{\tilde{M}_{ij}^2 + \delta \tilde{M}_{ij}^2, \tilde{B}_{ij} + \delta \tilde{B}_{ij}, \\
\tilde{A}_{ijk}, \tilde{M}_\lambda\} \quad \tilde{A}_{ijk} + \delta \tilde{A}_{ijk}, \tilde{M}_\lambda + \delta \tilde{M}_\lambda\} \\
\{u_{3ij}(0), v_3(0)\} \rightarrow \{u_{3ij}(0) + \delta u_{3ij}(0), v_3(0) + \delta v_3(0)\}
\end{aligned} \tag{46}$$

with real constants $\sqrt{Z_A}, \sqrt{Z_\lambda}, \sqrt{Z_c}, \sqrt{Z_{\phi_{ij}}}, \sqrt{Z_{\psi_{ij}}}, \delta g, \delta m_{ij}, \delta g_{ijk}, \delta \tilde{M}_{ij}^2, \delta \tilde{B}_{ij}, \delta \tilde{A}_{ijk}, \delta \tilde{M}_\lambda, \delta u_{3ij}(0), \delta v_3(0)$ that have to be compatible with the global symmetries.

Similarly, let δR be the following infinitesimal renormalization transformation:

$$\begin{aligned}
\delta R = \frac{1}{2} \delta Z_A \left[\int d^4x \left(A_a^\mu \frac{\delta}{\delta A_a^\mu} - Y_{A_a^\mu} \frac{\delta}{\delta Y_{A_a^\mu}} - B_a \frac{\delta}{\delta B_a} \right. \right. \\
\left. \left. - \bar{c}_a \frac{\delta}{\delta \bar{c}_a} \right) + 2\xi \frac{\partial}{\partial \xi} \right] + \frac{1}{2} \delta Z_\lambda \int d^4x \left(\lambda_a \frac{\delta}{\delta \lambda_a} + \bar{\lambda}_a \frac{\delta}{\delta \bar{\lambda}_a} \right. \\
\left. - Y_{\lambda_a} \frac{\delta}{\delta Y_{\lambda_a}} - Y_{\bar{\lambda}_a} \frac{\delta}{\delta Y_{\bar{\lambda}_a}} \right)
\end{aligned}$$

$$\begin{aligned}
+ \frac{1}{2} \delta Z_c \int d^4x \left(c \frac{\delta}{\delta c} - Y_c \frac{\delta}{\delta Y_c} \right) \\
+ \frac{1}{2} \delta Z_{\phi_{ij}} \int d^4x \left(\phi_j \frac{\delta}{\delta \phi_i} + \phi_j^\dagger \frac{\delta}{\delta \phi_i^\dagger} \right. \\
\left. - Y_{\phi_i} \frac{\delta}{\delta Y_{\phi_j}} - Y_{\phi_i^\dagger} \frac{\delta}{\delta Y_{\phi_j^\dagger}} \right) \\
+ \frac{1}{2} \delta Z_{\psi_{ij}} \int d^4x \left(\psi_j^\alpha \frac{\delta}{\delta \psi_i^\alpha} + \bar{\psi}_j \frac{\delta}{\delta \bar{\psi}_i} \right. \\
\left. - Y_{\psi_i^\alpha} \frac{\delta}{\delta Y_{\psi_j^\alpha}} - Y_{\bar{\psi}_i} \frac{\delta}{\delta Y_{\bar{\psi}_j}} \right) + \delta g \frac{\partial}{\partial g} + \delta m_{ij} \frac{\partial}{\partial m_{ij}} \\
+ \delta g_{ijk} \frac{\partial}{\partial g_{ijk}} + \delta \tilde{M}_{ij}^2 \frac{\partial}{\partial \tilde{M}_{ij}^2} + \delta \tilde{B}_{ij} \frac{\partial}{\partial \tilde{B}_{ij}} \\
+ \delta \tilde{A}_{ijk} \frac{\partial}{\partial \tilde{A}_{ijk}} + \delta \tilde{M}_\lambda \frac{\partial}{\partial \tilde{M}_\lambda} + \delta u_{3ij}(0) \frac{\partial}{\partial u_{3ij}(0)} \\
+ \delta v_3(0) \frac{\partial}{\partial v_3(0)} .
\end{aligned} \tag{47}$$

According to the results of Sect. 4.1 and using the identification

$$\begin{aligned}
\sqrt{Z_{\phi_{ij}}} &\rightarrow u_{1ij} , \\
\sqrt{Z_{\psi_{ij}}} &\rightarrow (u_1 u_2)_{ij} , \\
\sqrt{Z_A} &\rightarrow v_1 , \\
\sqrt{Z_\lambda} &\rightarrow v_1 v_2 ,
\end{aligned} \tag{48}$$

we see that both operators $R, \delta R$ are compatible with the symmetries. Suppose, Γ_{cl} is a classical solution of

$$\text{Sym}(\Gamma_{\text{cl}}) = 0.$$

Then $R\Gamma_{\text{cl}}$ is another solution:

$$\text{Sym}(R\Gamma_{\text{cl}}) = 0 , \tag{49}$$

and δR generates symmetric counterterms (compare (33)):

$$\begin{aligned}
\Gamma_{\text{sym}} &= \delta R \Gamma_{\text{cl}} \\
\Rightarrow \text{Sym}(\Gamma_{\text{cl}} + \zeta \Gamma_{\text{sym}}) &= 0 + \mathcal{O}(\zeta^2) .
\end{aligned} \tag{50}$$

Now we consider the symmetry identities and its classical solutions in the limit

$$a = \chi = 0, \text{ arbitrary.} \tag{51}$$

This limit is not identical with the physical limit (9) but better suited for our needs. In this limit the unwanted parameters do not appear but still the symmetry identities are restrictive enough.

Lemma: Let Γ_{cl} and Γ_{sym} denote a classical solution and an action for symmetric counterterms in the limit $a = \chi = 0$,

$$\text{Sym}(\Gamma_{\text{cl}})|_{a=\chi=0} = 0 , \tag{52}$$

$$\text{Sym}(\Gamma_{\text{cl}} + \zeta \Gamma_{\text{sym}})|_{a=\chi=0} = 0 + \mathcal{O}(\zeta^2) . \tag{53}$$

Then the most general form of Γ_{cl} , Γ_{sym} has to fulfil the relations

$$\Gamma_{\text{cl}}|_{a=\chi=0} = [R\Gamma_{\text{cl, canonical}}]|_{a=\chi=0} , \quad (54)$$

$$\Gamma_{\text{sym}}|_{a=\chi=0} = [\delta R\Gamma_{\text{cl, canonical}}]|_{a=\chi=0} , \quad (55)$$

with the operators R , δR defined in (46), (47).

Proof: The general classical solution of the symmetry identities (52), (53) can be obtained by a straightforward calculation. We write down a general ansatz, apply the symmetry identities and derive the necessary relations the coefficients in the ansatz have to satisfy. Although the calculation is lengthy, the announced results (54), (55) follow in a direct way.

We now give a short sketch of the calculation with emphasis on the main point, namely the restriction of the terms of $\mathcal{O}(\hat{f}, \hat{f}^\dagger)$. This sketch will also show why we have to use the limit (51) instead of (9) in the statement of the lemma.

The most general ansatz for Γ_{cl} can be decomposed according to the degree in a, χ, \hat{f} :

$$\Gamma_{\text{cl}} = \Gamma_0 + \Gamma_{\hat{f}, \text{lin}} + \Gamma_{\hat{f}, \text{rest}} + \Gamma_{\chi, \text{lin}} + \Gamma_{\text{rest}} , \quad (56)$$

where Γ_0 does not depend on a, χ, \hat{f} ; $\Gamma_{\hat{f}, \text{lin}}, \Gamma_{\hat{f}, \text{rest}}$ are linear and of higher degree in \hat{f} but do not depend on a, χ ; $\Gamma_{\chi, \text{lin}}$ is linear in χ and does not depend on a, \hat{f} , and Γ_{rest} contains the rest of the dependence on χ, \hat{f} , and the complete dependence on a .

Since all defining symmetry identities either do not change the degree in a, χ, \hat{f} or increase it, we obtain for Γ_0 :

$$0 = \text{Sym}(\Gamma)|_{a=\chi=\hat{f}=0} = \text{Sym}(\Gamma_0) , \quad (57)$$

thus Γ_0 is a classical solution of the defining symmetry identities in the case without soft breaking [6].

Next, the symmetry identities in (52) imply that $\Gamma_{\hat{f}, \text{lin}}$ is globally invariant and does not depend on B_a and ω^μ , and that

$$\begin{aligned} 0 &= S(\Gamma)|_{a=\chi=0, \text{linear in } \hat{f}} \\ &= s_{\Gamma_0}^0 \Gamma_{\hat{f}, \text{lin}} + S_\chi(\Gamma_{\chi, \text{lin}}) . \end{aligned} \quad (58)$$

Here $s_{\Gamma_0}^0$ is the linearized version of S_0 defined by

$$S_0(\Gamma_0 + \zeta\Gamma_1) = S_0(\Gamma_0) + \zeta s_{\Gamma_0}^0 \Gamma_1 + \mathcal{O}(\zeta^2) , \quad (59)$$

and

$$\begin{aligned} S_\chi(\Gamma) &= \int d^4x \left(s\chi^\alpha \frac{\delta\Gamma}{\delta\chi^\alpha} \Big|_{a=\chi=0} + s\bar{\chi}_{\dot{\alpha}} \frac{\delta\Gamma}{\delta\bar{\chi}_{\dot{\alpha}}} \Big|_{a=\chi=0} \right) \\ &= \int d^4x \left(\sqrt{2}\hat{f}\epsilon^\alpha \frac{\delta\Gamma}{\delta\chi^\alpha} \Big|_{a=\chi=0} \right. \\ &\quad \left. - \sqrt{2}\hat{f}^\dagger\bar{\epsilon}_{\dot{\alpha}} \frac{\delta\Gamma}{\delta\bar{\chi}_{\dot{\alpha}}} \Big|_{a=\chi=0} \right) . \end{aligned} \quad (60)$$

Due to the form of the operator S_χ we obtain

$$s_{\Gamma_0}^0 \Gamma_{\hat{f}, \text{lin}} = \mathcal{O}(\epsilon\hat{f}) + \mathcal{O}(\bar{\epsilon}\hat{f}^\dagger) . \quad (61)$$

Since on the physical fields $s_{\Gamma_0}^0$ acts as the BRS operator s up to field and parameter renormalizations, it is easy to see that the most general solution for $\Gamma_{\hat{f}, \text{lin}}$ that is compatible with the global symmetries is given by

$$\begin{aligned} \Gamma_{\hat{f}, \text{lin}} &= \hat{f} \left(\tilde{A}_{ijk}\phi_i\phi_j\phi_k + \tilde{B}_{ij}\phi_i\phi_j + \tilde{M}_\lambda\lambda_a\lambda_a \right. \\ &\quad \left. + u_{3ij}\sqrt{2}Y_{\psi_i}\epsilon\hat{f}\phi_j + v_3\sqrt{2}Y_{\bar{\chi}_a}\bar{\sigma}^\mu\epsilon A_{a\mu} \right) \\ &\quad + h.c. \end{aligned} \quad (62)$$

All these terms are accounted for in the operator R , (46).

This is the point where the limit (51) is important. If we had required only $\text{Sym}(\Gamma_{\text{cl}})|_{a=\chi=f=0}$ instead of (52), then we would have obtained only $\mathcal{O}(\epsilon) + \mathcal{O}(\bar{\epsilon})$ on the r.h.s. of (61), and in the solution to this equation non-GG terms $\phi\phi\phi^\dagger$ or $\psi\psi$ would have appeared.

The constraints on the remaining parts of Γ_{cl} can be worked out similarly.

5.2 Physical parameters

Once the symmetry identities are satisfied at a given order in the limit (51), there can still be divergent contributions which have to be absorbed by symmetric counterterms Γ_{sym} satisfying

$$\text{Sym}(\Gamma_{\text{cl}} + \zeta\Gamma_{\text{sym}})|_{a=\chi=0} = 0 + \mathcal{O}(\zeta^2) . \quad (63)$$

According to the lemma the most general form of Γ_{sym} is generated by the infinitesimal renormalization transformation

$$\Gamma_{\text{sym}}|_{a=\chi=0} = [\delta R\Gamma_{\text{cl}}]|_{a=\chi=0} . \quad (64)$$

This leads to the following hierarchy of the symmetric counterterms:

1. Counterterms appearing in physical processes, where not only $a = \chi = 0$, but also the external Y_i fields are set to zero:

$$\Gamma_{\text{sym}}|_{a=\chi=0, Y_i=0} . \quad (65)$$

This first class contains the counterterms to the field renormalization constants $Z_A, Z_\lambda, Z_c, Z_\phi, Z_\psi$ and the parameters $g, m_{ij}, g_{ijk}, \tilde{M}_{ij}^2, \tilde{B}_{ij}, \tilde{A}_{ijk}, \tilde{M}_\lambda$.

2. Additional counterterms appearing for $Y_i \neq 0$:

$$\Gamma_{\text{sym}}|_{a=\chi=0, Y_i \neq 0} . \quad (66)$$

This class contains precisely the counterterms to the u_3, v_3 parameters.

3. The rest of the counterterms appearing for a, χ arbitrary:

$$\Gamma_{\text{sym}}|_{a, \chi \neq 0, Y_i \neq 0} . \quad (67)$$

This class contains infinitely many independent counterterms.

The normalization conditions fixing the first, second and third set of counterterms we call *normalization conditions of the first, second and third class*, respectively.

The next theorem states how far we get using only the class-one-normalization conditions and leaving open the ones of the second and third class.

Theorem 1: Two solutions Γ_1 and Γ_2 of the same class-one-normalization conditions and of the symmetry identities in the limit (51),

$$\text{Sym}(\Gamma_2)|_{a=\chi=0} = \text{Sym}(\Gamma_1)|_{a=\chi=0} = 0, \quad (68)$$

can differ at most by local terms proportional to Y_ψ, Y_λ :

$$\begin{aligned} & (\Gamma_2 - \Gamma_1)|_{a=\chi=0} \\ &= \Delta_Y(u_{3ij}(0) + \delta u_{3ij}(0), v_3(0) + \delta v_3(0)) \\ &\equiv \int d^4x \left(-Y_{\psi_i\alpha} \sqrt{2}\epsilon^\alpha \hat{f}(u_{3ij}(0) + \delta u_{3ij}(0)) \phi_j \right. \\ &\quad \left. - Y_{\lambda\alpha} \sqrt{2}(v_3(0) + \delta v_3(0)) \hat{f}^\dagger \bar{\epsilon} \bar{\sigma}^{\mu\alpha} A_\mu \right) + h.c. \end{aligned} \quad (69)$$

Proof: Due to the lemma this holds at the tree level. To perform an inductive proof of this statement we suppose that we have at the order \hbar^{n-1} :

$$(\Gamma_2 - \Gamma_1)|_{a=\chi=0} = \Delta_Y(u_3^{(n-1)}, v_3^{(n-1)}) + \mathcal{O}(\hbar^n), \quad (70)$$

$$(\Gamma_{2,\text{ct}} - \Gamma_{1,\text{ct}})|_{a=\chi=0} = \Delta_Y(\delta u_3^{(n-1)}, \delta v_3^{(n-1)}) + \mathcal{O}(\hbar^n). \quad (71)$$

Then, at the next order *all* one-particle irreducible loop diagrams not involving a, χ are the same, regardless whether calculated according to the Feynman rules for Γ_1 or Γ_2 . This is true because even though the Feynman rules differ by the terms Δ_Y , these differences cannot contribute since they are linear in the propagating fields.

The difficult point is to prove that the counterterms of the order \hbar^n , denoted by $\Gamma_{1,\text{ct}}^{(n)}$ and $\Gamma_{2,\text{ct}}^{(n)}$, do not invalidate (70-71). We know

$$(\Gamma_2 - \Gamma_1)|_{a=\chi=0} = \Delta\Gamma_{\text{ct}}^{(n)} + \Delta_Y(u_3^{(n-1)}, v_3^{(n-1)}) + \mathcal{O}(\hbar^{n+1}), \quad (72)$$

$$\Delta\Gamma_{\text{ct}}^{(n)} = (\Gamma_{2,\text{ct}}^{(n)} - \Gamma_{1,\text{ct}}^{(n)})|_{a=\chi=0}. \quad (73)$$

Thus, taking into account the symmetry of Δ_Y and the fact that all symmetry identities except for the Slavnov-Taylor identity are linear and do not change the degree in a, χ , we obtain for these identities

$$\begin{aligned} 0 &= \text{Sym}(\Gamma_2)|_{a=\chi=0} \\ &= \text{Sym}(\Gamma_2|_{a=\chi=0}) \\ &= \text{Sym}\left(\Gamma_1|_{a=\chi=0} + \Delta\Gamma_{\text{ct}}^{(n)} + \Delta_Y(u_3^{(n-1)}, v_3^{(n-1)})\right) \\ &= 0 + \text{Sym}(\Delta\Gamma_{\text{ct}}^{(n)}). \end{aligned} \quad (74)$$

For the Slavnov-Taylor identity we obtain at the order \hbar^n (we use the operator S_χ defined in (60)):

$$\begin{aligned} 0 &= S(\Gamma_2)|_{a=\chi=0} \\ &= S_0(\Gamma_2|_{a=\chi=0}) + S_\chi(\Gamma_2) \\ &= S_0(\Gamma_1|_{a=\chi=0} + \Delta\Gamma_{\text{ct}}^{(n)} + \Delta_Y) + S_\chi(\Gamma_2) \\ &= S(\Gamma_1 + \Delta\Gamma_{\text{ct}}^{(n)} + \Delta_Y)|_{a=\chi=0} \\ &\quad + S_\chi(\Gamma_2 - (\Gamma_1 + \Delta\Gamma_{\text{ct}}^{(n)} + \Delta_Y)) \\ &= S(\Gamma_1 + \Delta\Gamma_{\text{ct}}^{(n)})|_{a=\chi=0} \\ &\quad + \int d^4x \left(\frac{\delta\Gamma_1 + \Delta\Gamma_{\text{ct}}^{(n)}}{\delta Y_i} \frac{\delta\Delta_Y}{\delta\varphi_i} \right. \\ &\quad \left. + \frac{\delta\Delta_Y}{\delta Y_i} \frac{\delta\Gamma_1 + \Delta\Gamma_{\text{ct}}^{(n)}}{\delta\varphi_i} \right) \Big|_{a=\chi=0} \\ &\quad + S_\chi(\Gamma_2 - (\Gamma_1 + \Delta\Gamma_{\text{ct}}^{(n)} + \Delta_Y)) \\ &= S(\Gamma_1 + \Delta\Gamma_{\text{ct}}^{(n)})|_{a=\chi=0} + \sqrt{2}(\epsilon^\alpha X_\alpha \hat{f} - \bar{\epsilon}_{\dot{\alpha}} \bar{X}^{\dot{\alpha}} \hat{f}^\dagger). \end{aligned} \quad (75)$$

The last two equations hold owing to the special form of Δ_Y with some suitably chosen functional X_α . Since Γ_1 satisfies the Slavnov-Taylor identity the first term of this result can be simplified using

$$S(\Gamma_1 + \Delta\Gamma_{\text{ct}}^{(n)}) = S(\Gamma_{1,\text{cl}} + \Delta\Gamma_{\text{ct}}^{(n)}) + \mathcal{O}(\hbar^{n+1}). \quad (76)$$

Therefore, both terms in the last line of (75) are local and power-counting renormalizable functionals of the order \hbar^n , and we can define a counterterm action

$$\Gamma_{\text{sym}} = \Delta\Gamma_{\text{ct}}^{(n)} + (\chi^\alpha X_\alpha + \bar{\chi}_{\dot{\alpha}} \bar{X}^{\dot{\alpha}}) \quad (77)$$

that satisfies

$$\begin{aligned} S(\Gamma_{1,\text{cl}} + \Gamma_{\text{sym}})|_{a=\chi=0} &= S(\Gamma_1 + \Delta\Gamma_{\text{ct}}^{(n)})|_{a=\chi=0} \\ &\quad + \sqrt{2}(\epsilon^\alpha X_\alpha \hat{f} - \bar{\epsilon}_{\dot{\alpha}} \bar{X}^{\dot{\alpha}} \hat{f}^\dagger) \\ &= 0 + \mathcal{O}(\hbar^{n+1}). \end{aligned} \quad (78)$$

Thus, Γ_{sym} is a symmetric counterterm in the sense of (63), and we obtain from the lemma:

$$\Gamma_{\text{sym}}|_{a=\chi=0} = [\delta R \Gamma_{1,\text{cl}}]|_{a=\chi=0} \quad (79)$$

On the other hand, by construction Γ_{sym} contains the relevant difference of Γ_1 and Γ_2 at the order \hbar^n :

$$(\Gamma_2 - \Gamma_1)|_{a=\chi=0} = \Gamma_{\text{sym}}|_{a=\chi=0} + \Delta_Y(u_3^{(n-1)}, v_3^{(n-1)}) + \mathcal{O}(\hbar^{n+1}). \quad (80)$$

Now, since $\Gamma_{1,2}$ satisfy the same class-one-normalization conditions, Γ_{sym} cannot contain any class-one-counterterms. Since these are the only counterterms that appear in the limit $a = \chi = Y_i = 0$, we obtain

$$\Gamma_{\text{sym}}|_{a=\chi=Y_i=0} = 0. \quad (81)$$

Owing to the concrete form of δR , this shows

$$\Delta\Gamma_{\text{ct}}^{(n)}|_{a=\chi=0} = \Gamma_{\text{sym}}|_{a=\chi=0} = \Delta_Y(\delta u_3^{(n)}, \delta v_3^{(n)}). \quad (82)$$

Together with (72) this demonstrates the validity of (70-71) at the next order, completing the induction.

5.3 Simplified symmetry identities at the quantum level

According to theorem 1, the parameters of class 2 and class 3 are irrelevant in physics respects. In this subsection, a complementary theorem is proven. This theorem 2 states that it is sufficient to establish the symmetry identities in the limit (51), where the infinitely many parameters of class 3 do not appear at all. This implies that the class 3 parameters can be completely ignored in practice. The two parameters u_3, v_3 of class 2 are also unphysical, but they do appear in the limit (51).

At the classical level, this is a direct consequence of the Lemma in Sect. 5.1 together with (49), (50): Any classical solution Γ_{cl} of the symmetry identities (52) is equivalent in physics respects to a solution $[R\Gamma_{\text{cl}}, \text{canonical}]$ of the full symmetry identities. In this subsection the theorem is extended to the quantum level. The statement of the theorem and its proof at the quantum level is divided into two parts – the existence of a solution to the symmetry identities in the limit (51) and its extension to a full solution.

5.3.1 Existence of a solution

Theorem 2a: Suppose Γ is a solution of the symmetry identities in the limit (51) up to the order \hbar^{n-1} ,

$$\text{Sym}(\Gamma)|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^n), \quad (83)$$

and Γ^{exact} is an extension that solves the full symmetry identities,

$$\text{Sym}(\Gamma^{\text{exact}}) = 0 + \mathcal{O}(\hbar^n), \quad (84)$$

$$(\Gamma^{\text{exact}} - \Gamma)|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^n). \quad (85)$$

Then we claim that $\Gamma, \Gamma^{\text{exact}}$ can be renormalized in such a way that the (83-85) are maintained at the next order \hbar^n .

Proof: Since we assume the absence of anomalies, Γ^{exact} can be renormalized in such a way that

$$\text{Sym}(\Gamma^{\text{exact}}) = 0 + \mathcal{O}(\hbar^{n+1}). \quad (86)$$

Since the Feynman rules of the order \hbar^n defined by Γ^{exact} and Γ differ only in terms $\sim a, \chi$, all loop diagrams contributing to $\Gamma^{\text{exact}}|_{a=\chi=0}$ and $\Gamma|_{a=\chi=0}$ are equal at this order. Thus, adding appropriate $\mathcal{O}(\hbar^n)$ counterterms to Γ we obtain

$$(\Gamma^{\text{exact}} - \Gamma)|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^{n+1}). \quad (87)$$

However, Γ does not yet satisfy the Slavnov-Taylor identity at this order. Indeed, neglecting terms of the order \hbar^{n+1} we obtain

$$\begin{aligned} S(\Gamma)|_{a=\chi=0} &= S_0(\Gamma|_{a=\chi=0}) + S_\chi(\Gamma) \\ &= S_0(\Gamma^{\text{exact}}|_{a=\chi=0}) + S_\chi(\Gamma) \\ &= S(\Gamma^{\text{exact}})|_{a=\chi=0} + S_\chi(\Gamma - \Gamma^{\text{exact}}) \\ &= S_\chi(\Gamma - \Gamma^{\text{exact}}) \\ &= \hbar^n \Delta. \end{aligned} \quad (88)$$

Owing to the quantum action principle [16], the lowest order of Δ is a local and power-counting renormalizable functional, and owing to the form of S_χ it takes the form

$$\Delta = \int \sqrt{2}\epsilon^\alpha X_\alpha \hat{f} - \sqrt{2}\bar{\epsilon}_{\dot{\alpha}} \bar{X}^{\dot{\alpha}} \hat{f}^\dagger + \mathcal{O}(\hbar). \quad (89)$$

Hence, adding the counterterms

$$\Gamma \rightarrow \Gamma - \int \hbar^n (\chi^\alpha X_\alpha + \bar{\chi}_{\dot{\alpha}} \bar{X}^{\dot{\alpha}}) \quad (90)$$

restores the Slavnov-Taylor identity $S(\Gamma)|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^{n+1})$ without interfering with (87). All further symmetry identities are linear and homogeneous in a, χ . Therefore, Γ satisfies these identities, too, and we obtain

$$\text{Sym}(\Gamma)|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^{n+1}). \quad (91)$$

This was to be shown.

5.3.2 Extension to a full solution

Theorem 2b: Let Γ be a solution to the symmetry identities in the limit $a = \chi = 0$,

$$\text{Sym}(\Gamma)|_{a=\chi=0} = 0. \quad (92)$$

Then there exists an extension to a full solution Γ^{exact} satisfying

$$\text{Sym}(\Gamma^{\text{exact}}) = 0, \quad (93)$$

$$(\Gamma^{\text{exact}} - \Gamma)|_{a=\chi=0} = 0. \quad (94)$$

Proof: Due to the lemma there is a classical solution $\Gamma_{\text{cl}}^{\text{exact}}$ satisfying (93-94). Now suppose the same is true at the order \hbar^{n-1} , that is there exists an effective action Γ^{exact} satisfying

$$\text{Sym}(\Gamma^{\text{exact}}) = 0 + \mathcal{O}(\hbar^n), \quad (95)$$

$$(\Gamma^{\text{exact}} - \Gamma)|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^n). \quad (96)$$

Then, according to theorem 2a there are $\mathcal{O}(\hbar^n)$ counterterms yielding $\tilde{\Gamma} = \Gamma + \mathcal{O}(\hbar^n)$, $\tilde{\Gamma}^{\text{exact}} = \Gamma^{\text{exact}} + \mathcal{O}(\hbar^n)$ such that

$$\text{Sym}(\tilde{\Gamma})|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^{n+1}), \quad (97)$$

$$\text{Sym}(\tilde{\Gamma}^{\text{exact}}) = 0 + \mathcal{O}(\hbar^{n+1}), \quad (98)$$

$$(\tilde{\Gamma}^{\text{exact}} - \tilde{\Gamma})|_{a=\chi=0} = 0 + \mathcal{O}(\hbar^{n+1}). \quad (99)$$

However, due to (92), (97) the difference $\tilde{\Gamma} - \Gamma$ has to be a symmetric counterterm as defined in (53). Hence, it has the form

$$(\Gamma - \tilde{\Gamma})|_{a=\chi=0} = [\delta R\Gamma_{\text{cl}}]|_{a=\chi=0}. \quad (100)$$

Therefore, $\Gamma^{\text{exact}} = \tilde{\Gamma}^{\text{exact}} + \delta R\Gamma_{\text{cl}}^{\text{exact}}$ has the desired properties

$$\begin{aligned} \text{Sym}(\Gamma^{\text{exact}}) &= \text{Sym}(\tilde{\Gamma}^{\text{exact}} + \delta R\Gamma_{\text{cl}}^{\text{exact}}) \\ &= 0 + \mathcal{O}(\hbar^{n+1}), \end{aligned} \quad (101)$$

$$\begin{aligned} (\Gamma^{\text{exact}} - \Gamma)|_{a=\chi=0} &= (\tilde{\Gamma}^{\text{exact}} - \tilde{\Gamma})|_{a=\chi=0} \\ &= 0 + \mathcal{O}(\hbar^{n+1}). \end{aligned} \quad (102)$$

This completes the induction.

6 Alternative approach

The first Slavnov-Taylor identity for softly broken supersymmetric gauge theories was presented in [3]. In this construction the absence of anomalies could be nicely shown, but there appeared new kinds of parameters whose physical meaning remained unclear. As shown in Sect. 5, in our approach this problem could be solved. In this section a brief comparison of both approaches is given.

Basically, in both approaches the soft breaking is introduced via external fields with definite BRS transformation rules. These transformation rules contain a constant shift that yields the soft parameters in the limit of vanishing external fields.

The main difference concerns the underlying intuition and consequently the external field content⁶: In [3], the soft breaking terms are not introduced as couplings to a multiplet (a, χ, \hat{f}) that transforms as a chiral supermultiplet but as couplings to a BRS doublet (u, \hat{v}) where⁷

$$su = \hat{v} - i\omega^\nu \partial_\nu u, \quad (103)$$

$$sv = 2i\epsilon\sigma^\nu \bar{\epsilon} \partial_\nu u - i\omega^\nu \partial_\nu v, \quad (104)$$

$$\hat{v}(x) = v(x) + \kappa. \quad (105)$$

The main benefit of this structure is that the cohomological sector of the theory is not altered compared to the case without soft breaking. This allows a straightforward proof of the absence of anomalies.

Contrary to the case of (a, χ, \hat{f}) , however, the BRS transformations of u and v cannot be interpreted as supersymmetry transformations where simply the transformation parameter has been promoted to a ghost. Moreover, u and v are two scalar fields and therefore cannot form a supersymmetry multiplet. Correspondingly, the restriction of the breaking terms to the ones of the GG-class is done by requiring R -invariance with specially chosen R -weights. In [3], requiring supersymmetry alone would not suffice to forbid non-GG terms (see Sect. 2.2). On the one hand, this opens a way to perform the renormalization of theories with arbitrary supersymmetry breaking. But on the other hand the emphasized role of R -invariance might obstruct a deeper understanding of softly broken supersymmetry and its influence on typical consequences of supersymmetry like non-renormalization properties.

In the limit of vanishing external fields, the classical action in both approaches reduces to the same soft breaking action but for non-vanishing external fields in both cases new parameters appear: in our case the ones discussed

⁶ One further difference concerns the supersymmetric mass terms which are also introduced via external fields in [3]. This is done in order not to violate R -invariance because the R -weights of the chiral fields are fixed to $n_i = \frac{2}{3}$ (translated to our convention) in accordance with the R -part of the supercurrent. In our case the R -weights are assumed to be chosen in such a way that the mass terms are invariant and therefore we do not need such an external field multiplet

⁷ The equations are translated to our conventions. In particular, in [3] there is also an R -transformation part in the BRS transformations, which is neglected here

in Sect. 4.1, in the case of [3] for instance the parameters ρ_2, ρ_4 that appear in the terms

$$\Gamma_{2,4} = \int d^4x \left(\rho_{2ab} Y_{\psi_b \alpha} \epsilon^\alpha (\hat{v} \phi_a - \sqrt{2} u \epsilon \psi_a) + \rho_{4ab} \hat{v} \bar{u} \epsilon \psi_a \phi_b^\dagger \right) + \dots \quad (106)$$

The main reason why the approach of [3] cannot be used directly in phenomenological applications is that the physical meaning of these parameters is not obvious. In particular, a theorem showing whether these parameters are irrelevant for physical quantities or not – analogous to Sect. 5.2 – is lacking.

In spite of these differences, there is a remarkable relation between both approaches. First of all, the quantum numbers of \hat{v} and \hat{f} are equal, and second we can combine the supersymmetry ghost and u to a spinor (ϵu) that has the same quantum numbers as χ . Hence, we can identify

$$\begin{aligned} a &\rightarrow 0, \\ \chi^\alpha &\rightarrow \epsilon^\alpha u, \\ \sqrt{2} \hat{f} &\rightarrow \hat{v}. \end{aligned} \quad (107)$$

Furthermore, this correspondence even holds for the BRS transformations:

$$\begin{aligned} sa &\rightarrow \sqrt{2} \epsilon \epsilon u = 0, \\ s\chi^\alpha &\rightarrow \sqrt{2} \epsilon^\alpha \hat{v} - i\omega^\nu \partial_\nu \epsilon^\alpha u = s\epsilon^\alpha u, \\ s\sqrt{2} \hat{f} &\rightarrow 2i\bar{\epsilon} \sigma^\nu \partial_\nu \epsilon u - i\omega^\nu \partial_\nu \hat{v} = s\hat{v}. \end{aligned} \quad (108)$$

Here we have used $\epsilon^\alpha \epsilon_\alpha = 0$, which holds since ϵ is bosonic. Thus, u and \hat{v} may be regarded as a part of our chiral multiplet (a, χ, \hat{f}) . And there is a natural identification in our framework of terms like the ρ_2 -term in (106), where u comes always in combination with ϵ . In fact, this term has the same structure as the u_3 -term in (36) with $u_3 \rightarrow -\rho_2$ when (107) is used.

However, in the classical action of [3] there are also terms where u appears without an accompanying ϵ or \bar{u} without accompanying $\bar{\epsilon}$, such as the ρ_4 -term in (106). These terms have no correspondence in our framework. On the other hand, of course our terms depending on the a field have no correspondence in [3]. Therefore both frameworks are really different and independent of each other.

7 Conclusions

In this article we have performed the renormalization of supersymmetric Yang-Mills theories with soft supersymmetry-breaking terms of the GG class. These terms are introduced in a supersymmetric way via an external chiral multiplet, allowing a construction that parallels the one without soft breaking.

This construction is afflicted by a problem, since in the course of the renormalization, an unconstrained number of additional parameters appear. However, in Sect. 5

it is shown that these parameters are irrelevant in physics respects. Even better than gauge parameters they do not influence any vertex functions that occur in physical S-matrix elements; and neither at the classical nor at the quantum level it is necessary to calculate the part of the Lagrangian and the counterterms involving those additional parameters.

For practical calculations of physical processes the theorems in Sect. 5 imply, first, that the symmetry identities need to be established only in the limit (51),

$$\text{Sym}(\Gamma)|_{a=\chi=0} = 0.$$

And second, renormalization of the fields and parameters appearing in the relevant part of the classical action suffices to cancel the divergences.

Since the supersymmetric extensions of the standard model like the minimal one (MSSM) involve soft breaking, our results provide an important building block for the renormalization of these kind of models.

The impossibility to accommodate non-GG breaking terms in the framework with spurion fields, where breaking terms are introduced via a coupling to a supermultiplet, shows that GG terms are a renormalizable subclass of all breaking. That these terms have even special properties under renormalization, as seen in explicit one-loop calculations and different approaches to their renormalization group coefficients [7–10], cannot be concluded by using the present formalism. As shown for the Abelian case in [10], the present formalism provides the correct starting point for this purpose, but it has to be enhanced by a deeper characterization of the symmetries.

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Appendix

A Conventions

2-Spinor indices and scalar products:

$$\psi\chi = \psi^\alpha\chi_\alpha, \quad \bar{\psi}\bar{\chi} = \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}, \quad (109)$$

σ matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (110)$$

$$\sigma_{\alpha\dot{\alpha}}^\mu = (1, \sigma^k)_{\alpha\dot{\alpha}}, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha} = (1, -\sigma^k)^{\dot{\alpha}\alpha}, \quad (111)$$

$$(\sigma^{\mu\nu})_\alpha{}^\beta = \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)_\alpha{}^\beta, \quad (112)$$

$$(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} = \frac{i}{2}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu)^{\dot{\alpha}}{}_{\dot{\beta}}. \quad (113)$$

Complex conjugation:

$$(\psi\theta)^\dagger = \bar{\theta}\bar{\psi}, \quad (114)$$

$$(\psi\sigma^\mu\bar{\theta})^\dagger = \theta\sigma^\mu\bar{\psi}, \quad (115)$$

$$(\psi\sigma^{\mu\nu}\theta)^\dagger = \bar{\theta}\bar{\sigma}^{\mu\nu}\bar{\psi}. \quad (116)$$

Derivatives:

$$\frac{\partial}{\partial\theta^\alpha}\theta^\beta = \delta_\alpha{}^\beta, \quad \frac{\partial}{\partial\theta_\alpha}\theta_\beta = -\delta_\beta{}^\alpha, \quad (117)$$

$$\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}\bar{\theta}_{\dot{\beta}} = \delta_{\dot{\alpha}}{}^{\dot{\beta}}, \quad \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\bar{\theta}^{\dot{\beta}} = -\delta^{\dot{\beta}}{}_{\dot{\alpha}}. \quad (118)$$

B BRS transformations

On the physical fields (i.e. fields carrying no ghost number) the BRS transformations are the sum of gauge and supersymmetry transformations and translations, where the transformation parameters have been promoted to the ghost fields:

$$sA_\mu = \partial_\mu c - ig[c, A_\mu] + i\epsilon\sigma_\mu\bar{\lambda} - i\lambda\sigma_\mu\bar{\epsilon} - i\omega^\nu\partial_\nu A_\mu, \quad (119)$$

$$s\lambda^\alpha = -ig\{c, \lambda^\alpha\} + \frac{i}{2}(\epsilon\sigma^{\rho\sigma})^\alpha F_{\rho\sigma} + i\epsilon^\alpha D - i\omega^\nu\partial_\nu\lambda^\alpha, \quad (120)$$

$$s\bar{\lambda}_{\dot{\alpha}} = -ig\{c, \bar{\lambda}_{\dot{\alpha}}\} - \frac{i}{2}(\bar{\epsilon}\bar{\sigma}^{\rho\sigma})_{\dot{\alpha}} F_{\rho\sigma} + i\bar{\epsilon}_{\dot{\alpha}} D - i\omega^\nu\partial_\nu\bar{\lambda}_{\dot{\alpha}}, \quad (121)$$

$$s\phi_i = -igc\phi_i + \sqrt{2}\epsilon\psi_i - i\omega^\nu\partial_\nu\phi_i, \quad (122)$$

$$s\phi_i^\dagger = +ig(\phi^\dagger c)_i + \sqrt{2}\bar{\psi}_i\bar{\epsilon} - i\omega^\nu\partial_\nu\phi_i^\dagger, \quad (123)$$

$$s\psi_i^\alpha = -igc\psi_i^\alpha + \sqrt{2}\epsilon^\alpha F_i - \sqrt{2}i(\bar{\epsilon}\bar{\sigma}^\mu)^\alpha D_\mu\phi_i - i\omega^\nu\partial_\nu\psi_i^\alpha, \quad (124)$$

$$s\bar{\psi}_{i\dot{\alpha}} = -ig(\bar{\psi}_{\dot{\alpha}}c)_i - \sqrt{2}\bar{\epsilon}_{\dot{\alpha}} F_i^\dagger + \sqrt{2}i(\epsilon\sigma^\mu)_{\dot{\alpha}}(D_\mu\phi_i)^\dagger - i\omega^\nu\partial_\nu\bar{\psi}_{i\dot{\alpha}}, \quad (125)$$

$$sa = \sqrt{2}\epsilon\chi - i\omega^\nu\partial_\nu a, \quad (126)$$

$$sa^\dagger = \sqrt{2}\bar{\chi}\bar{\epsilon} - i\omega^\nu\partial_\nu a^\dagger, \quad (127)$$

$$s\chi^\alpha = \sqrt{2}\epsilon^\alpha\hat{f} - \sqrt{2}i(\bar{\epsilon}\bar{\sigma}^\mu)^\alpha\partial_\mu a - i\omega^\nu\partial_\nu\chi^\alpha, \quad (128)$$

$$s\bar{\chi}_{\dot{\alpha}} = -\sqrt{2}\bar{\epsilon}_{\dot{\alpha}}\hat{f}^\dagger + \sqrt{2}i(\epsilon\sigma^\mu)_{\dot{\alpha}}\partial_\mu a^\dagger - i\omega^\nu\partial_\nu\bar{\chi}_{\dot{\alpha}}, \quad (129)$$

$$sf = \sqrt{2}i\bar{\epsilon}\bar{\sigma}^\mu\partial_\mu\chi - i\omega^\nu\partial_\nu f, \quad (130)$$

$$sf^\dagger = -\sqrt{2}i\partial_\mu\bar{\chi}\bar{\sigma}^\mu\epsilon - i\omega^\nu\partial_\nu f^\dagger. \quad (131)$$

Here we have used $A_\mu = T^a A_{a\mu}$ and similar for $\lambda, \bar{\lambda}, F_{\rho\sigma}, D, c, \bar{c}, B$. Again, the auxiliary fields D and F_i, F_i^\dagger are understood to be eliminated by their equations of motion.

The various (anti)commutation relations of the transformations are encoded in the nilpotency equation

$$s^2 = 0 + \text{field equations} \quad (132)$$

if the BRS transformations of the ghosts are given by the structure constants of the algebra and the ghosts have the opposite statistics as required by the spin-statistics theorem [14]:

$$sc = -igc^2 + 2i\epsilon\sigma^\nu\bar{\epsilon}A_\nu - i\omega^\nu\partial_\nu c, \quad (133)$$

$$s\epsilon^\alpha = 0, \quad (134)$$

$$s\bar{\epsilon}_{\dot{\alpha}} = 0, \quad (135)$$

$$s\omega^\nu = 2\epsilon\sigma^\nu\bar{\epsilon}. \quad (136)$$

The BRS transformations of the antighosts and B fields read

$$s\bar{c}_a = B_a - i\omega^\nu \partial_\nu \bar{c}_a , \quad (137)$$

$$sB_a = 2i\epsilon\sigma^\nu \bar{c} \partial_\nu \bar{c}_a - i\omega^\nu \partial_\nu B_a . \quad (138)$$

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